Note on Photographs of Comet d 1902 Giacobini obtained with the 30-inch Reflector at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

Seven photographs of this comet have been obtained with the 30-inch reflector on six nights, with exposures varying from thirty minutes to five minutes. In the case of three photographs the driving of the telescope was corrected to allow for the comet's motion, but the remaining photos were taken with the telescope guided on the stars, the comet's motion in ten minutes being very small. The comet is well shown with an exposure of ten minutes, the photographic image being of stellar character, and capable of very exact measurement.

The following is a list of the photographs obtained up to the present time :—

Date.	Exposure.	Remarks.
1902.		
Dec. 29	30^m	Comet's motion in R.A. and Decl. corrected.
,, 30	10^m, 15^m	Comet's motion in Decl. corrected.
,, 31	10^m, $7\frac{1}{2}^m$, 10^m, 5^m	,, ,,
1903.		
Jan. 2	10^m, 10^m, 8^m	Telescope guided on the stars.
,, 3	10^m, 10^m, 10^m	,, ,,
,, 7	15^m	,, ,,
,, 7	15^m	,, ,,
,, 8	10^m, 10^m	,, ,,

Royal Observatory, Greenwich :
1903 *January* 9.

Note on Plate Constants. By F. W. Dyson, M.A., F.R.S.

In a paper by the Astronomer Royal and myself (*Monthly Notices*, vol. lvi. pp. 114–134) the reduction of a number of astrographic plates according to the formulæ and methods given by Professor Turner (*Monthly Notices*, vol. liv.) was considered. It was shown that the constants a and e computed in this way gave discordances with the values of these constants calculated so as to correct for the scale of the telescope, and for the differential effects of aberration and refraction of a magnitude which might be expected from the errors of the right ascensions and declinations of the reference stars. Similarly the constants b and d, which are mainly caused by an orientation error of the plate, showed differences of the magnitude which would arise from the errors of the reference stars. It was considered, therefore, that it would be better to use calculated values of a and e,

and a value of the orientation error from the mean of the *b* and *d* results, and thus use the data of the plate to furnish only three constants—namely, the orientation and the errors of the centre of the plate in the two directions. This was a modification of Professor Turner's method, in which six independent constants were derived from the data of the plate itself. The grounds for the use of six constants are that these will tend to correct for possible errors of distortion, whether optical or arising during the drying of the gelatine, tilt of the plate, &c. The conclusion which was arrived at by the Astronomer Royal and myself was that the uncertainties of the position of the reference stars were sufficient to account for the differences of the plate constants from their theoretical values.

The six plate constants have been computed by Professor Turner's method for 508 of the Greenwich astrographic plates. For 376 of these the right ascensions and declinations of the Helsingfors and Christiania Catalogues have been used. These stars are being reobserved at Greenwich; and, although not more than two observations have been generally obtained, it seemed of interest to recompute with the positions of the reference stars the plate constants for those plates which showed the greatest discordances.

The following table gives the results of the two determinations (the unit being in the fifth decimal place, *i.e.* −99 is −·00099, &c.) :—

TABLE I.

Plate Constants.

No. of Plate.	No. of Stars.	Old Determination.				New Determination.			
		a.	*e.*	*b.*	*d.*	*a.*	*e.*	*b.*	*d.*
336	21	− 57	− 41	+133	−180	− 85	− 65	+166	−159
807	16	− 48	−110	+296	−306	− 61	− 66	+279	−314
862	11	− 6	− 57	+281	−320	− 56	− 79	+305	−282
938	14	− 53	−126	+105	−108	− 68	− 80	+101	−124
941	12	− 27	− 90	+ 80	− 60	− 60	− 71	+ 75	− 69
1610	18	− 99	− 68	+120	−190	− 57	− 76	+137	−158
1762	17	− 96	− 40	+407	−428	− 87	− 74	+373	−414
2042	22	− 77	−137	−107	+129	− 92	− 81	−109	+138
2044	21	− 95	− 77	+ 36	− 98	− 99	− 77	+ 40	− 65
2048	28	− 54	− 32	+ 45	− 10	− 59	− 46	+ 20	− 4
2328	17	−103	− 37	+122	−130	−105	− 95	+138	−159
2568	25	− 30	− 85	+102	−106	− 55	− 85	+102	−106
2685	20	− 66	− 82	+220	−282	− 78	− 81	+235	−244
2859	41	− 74	− 77	+242	−308	− 70	− 76	+293	−300
2981	15	− 35	−113	+370	−338	− 76	− 66	+373	−371
2995	9	− 57	− 33	+294	−303	− 67	− 63	+309	−332
4807	18	− 67	− 28	+ 77	− 71	− 78	− 76	+ 19	− 31

It will be seen that all the large discordances in the values of a and e, whose theoretical value is about -00070 (varying principally owing to aberration from -00064 to -00076) are removed.

The discordances between the plate constants are so much diminished by improved right ascensions and declinations of the reference stars that it seems reasonable to look upon the uncertainties in the positions of these stars as the main source of error, and to conclude that any real variations of the constants due to distortion, &c., are less than the accidental errors of their determination. This is at any rate the case unless the reference stars are extremely well observed; and less error will be introduced by neglecting the small possible effects of distortion, tilt, &c., than by introducing the accidental errors which are involved in the determination of three additional constants.

The large difference in the values of b and d in the last plate, in addition to the change in the value of e, made it of interest to compute the final residuals with the adopted constants $a = e = -{\cdot}00075$ $b = -d = +25$. The following table gives the right ascensions originally used, the right ascensions and declinations from recent Greenwich observations, and the differences between Greenwich and Christiania, and between the Greenwich meridian observations and the photographs. The numbers in the third column give the number of observations at Greenwich.

TABLE II.

(*Assumed right ascensions and declinations of reference stars on Plate* 4807 *in the first and second determinations of plate constants, and the final residuals with the adopted constants* $a = e = -{\cdot}00075$ $b = -d = +{\cdot}00025$.)

No. in Chr. Cat.	Position from Greenwich Observations. R.A.		Decl.	No. of Obs.	Corrections to Christiania. R.A.		Decl.	Final Residuals on Photo. R.A.	Decl.
	h m s	°	′ ″		s	″	″	″	″
1567	9 57 51·93	68	4 37·4	3	−·01 =	−0·0	+2·1	+0·5	−0·4
51	49 12·97		14 12·5	2	+·13	+0·7	+2·2	+0·5	+0·2
66	57 34·62		23 5·8	2	−·07	−0·6	+0·3	−0·5	−1·1
41	45 30·49		38 14·5	2	+·38	+2·1	−0·2	−0·7	+0·4
32	39 11·82		56 50·1	2	−·14	−0·7	+0·5	−0·5	+0·7
52	49 18·74		57 19·3	2	−·08	−0·4	−2·0	+0·3	+0·4
79	10 1 33·70		59 9·8	1	−·11	−0·6	+1·2	+1·2	−0·3
68	9 58 23·15	69	8 57·9	2	−·07	−0·4	+1·5	+0·3	0·0
56	53 28·74		11 52·4	3	−·08	−0·5	−0·1	+0·3	−0·1
77	10 0 56·22		11 29·9	2	−·21	−1·1	+0·8	−0·7	−0·2
80	1 37·80		9 13·9	1	−·20	−1·1	−0·2	−1·4	−0·2
59	9 54 53·81		16 3·3	3	−·34	−1·7	+1·0	−0·6	+1·2
44	46 47·25		24 42·8	1	−·45	−2·4	0·0	0·0	−0·4
45	46 49·08		24 42·6	1	−·58	−3·0	−1·2	+0·5	−0·2

No. in Chr. Cat.	Position from Greenwich Observations. R.A.	Decl.	No. of Obs.	Corrections to Christiania. R.A.		Decl.	Final Residuals on Photo. R.A.	Decl.
	h m s	° ′ ″		s	″	″	″	″
46	46 49·97	22 21·1	5	+·02	+0·1	−1·5	+0·8	+0·1
50	48 47·58	22 28·0	2	+·20	+1·1	−0·2	−0·4	−0·6
58	54 20·45	31 16·3	4	−·27	−1·4	+0·6	−0·1	+0·1
42	46 8·34	55 21·7	1	−·61	−4·2	−4·0	+0·5	−0·8

Taking this plate, which was chosen quite at random, as a fair sample of the results which will be given by the Greenwich astrographic work, the smallness of the residuals is extremely satisfactory.

In R.A. the mean discordance is $\pm{\cdot}0018 = \pm 0''{\cdot}54$
In Decl. " " " " $\pm{\cdot}0014 = \pm 0''{\cdot}41$

The corresponding probable errors are $\pm 0''{\cdot}46$ and $\pm 0''{\cdot}35$.

In the introduction to the Greenwich Catalogue for 1890 the probable accidental error of an observation of right ascension is given as $\pm^{s}{\cdot}033$ ($= \pm''{\cdot}50$), and between the limits of Decl. of the Greenwich Zone the probable accidental error of an observation of Decl. is $\pm 0''{\cdot}46$. Each of the reference stars on this plate has been observed about twice, so that the resulting R.A. and Decl. may be expected to have probable errors of about $\pm 0''{\cdot}35$ and $\pm 0''{\cdot}32$.

Comparing these with the figures $\pm 0''{\cdot}46$ and $\pm 0''{\cdot}35$, it is clear that the errors of the photographic results are extremely small.

The above table, as well as Table I., shows the advantage which has resulted from the re-observation of the reference stars with the transit circle. With five observations of each reference star it would seem the uncertainties of the plate constants, which are the principal source of the error in the deduction of R.A. and Decl. from the astrographic photographs, will be sufficiently reduced.

Returning now to the computation of the plate constants, there is a considerable simplification of the work when a and e are known beforehand. The quantities $x-ax$ are tabulated in order of y and equated to $by+c$. The solution of these equations, which should be done by least squares with approximate weights when the number of stars is small or when the stars are badly distributed, gives b and c. Similarly the quantities $y-ey$ are arranged in order of x and equated to $dx+f$, and the equations solved for d and f. The mean of b and $-d$, with a possible correction of ·00001 when the plate is taken one hour from the meridian, is adopted. The advantage of the arrangement in order of y and x is that any mistakes which may have been overlooked are likely to be detected before the equations are solved.

A Graphical Method of Applying to Photographic Measures the Terms of the Second Order in the Differential Refraction. Arthur R. Hinks, M.A.

1. The great and well-known advantage of Turner's methods of reducing in rectangular coordinates the measures made upon a photograph depends upon the fact that all the various small differences, due to refraction, aberration, orientation, centering, &c., by which the measures of stars on an actual plate differ from what they would be on an ideal "standard" plate are usually linear functions of the coordinates, and their value can be deduced *en bloc* by comparing the measured coordinates of known stars with their computed coordinates on the ideal plate. So soon as any of the corrections necessary begin to involve terms of the second order in x and y, the measured coordinates, the simplicity of the method is lost, but may be restored by finding separately and applying to the measures such parts of the whole reductions as are of the second order in x and y; when this is done the main part of the reduction proceeds as before.

2. The only parts of the reductions which are likely to involve second-order terms are the differential refractions. In the reduction of the photographs of *Eros*, many of which were necessarily taken at considerable zenith distances, these terms occasionally amount to one- or two-tenths of a second of arc; it is therefore necessary to take account of them before proceeding to form the equations and make the solution for the ordinary six constant reductions. But since nothing is more aggravating than having to compute for each star a number of small quantities which when added together amount generally to not more than one or two units in the last place of decimals employed, I tried various ways of computing these small terms graphically, and have succeeded in constructing three diagrams by means of which the second-order refraction terms can be found in a few minutes for any plate taken anywhere in the world. It seems possible, then, that these diagrams may be found of general use.

3. The expressions for the complete effect of refraction have been given by Professor Turner (*Monthly Notices*, lvii. 1897, p. 133). It is there shown that if (x, y) be the coordinates of an unrefracted star, and β the constant of refraction, the effect of refraction is to displace the star image on the plate to the point whose coordinates are

$$x+t(\mathrm{X}-x),\ y+t(\mathrm{Y}-y)$$

where

$$t=\beta\frac{1+x^2+y^2}{1+x\mathrm{X}+y\mathrm{Y}+\beta\{x(x-\mathrm{X})+y(y-\mathrm{Y})\}}$$